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Pure Circular Motion with Non-Angular Variables in One-Dimensional Motion Physics Problems

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This paper aims to introduce a third section in standard physics books consisting of pure circular motion expressed as straight-line motion with the particle travelling along the positive x-axis. This can greatly help students visualize the more difficult general curvilinear motion seen in their sophomore year dynamics course.

In standard algebra and calculus-based college physics textbooks, chapter two normally introduces one-dimensional motion for a particle with the following equations v = dx/dt and a = dv/dt. The kinematic equations with constant acceleration a_x are then derived for motion along the x-axis. Then a second section is introduced for motion along the y-axis for freely falling objects in which the constant acceleration is $a_y = g = 9.81\ m/s^2$. This pedagogical paper suggests that a third section be included for a particle travelling in pure circular motion. With the introduction of one new formula for Radial Acceleration, the standard kinematic equations can be used. The standard kinematic equations will be defined with tangential velocity and acceleration.

This paper proposes to model particles travelling along the circumference of a circle with radius R relating displacement x as multiples of n Straight-Line segments of length C = 2πR along a horizontal axis with markings of 0C, 1C, 2C, 3C etc. Once the formula for radial acceleration is derived, the following axes will also be defined. The positive x and positive Tangential Velocity axes. For these problems we simplify things and will consider only counterclockwise motion. This direction will be considered positive. In addition, a moving particle will not come to rest and reverse direction. The positive x-axis can then be related to the positive circumference axis through the equation x = nC in which n is the number of circumference lengths travelled by the particle or the number of times the particle travelled around its circular path. Lastly, the tangential acceleration axis which will have both positive and negative acceleration values and perpendicular to that axis, the positive radial acceleration axis will be introduced. The positive radial acceleration axis will point to the center of the circle.

Both uniform and non-uniform circular motion will be discussed. The difference in these pure circular motion problems versus one-dimensional motion along the x or y axes is that the resultant acceleration vector will have two components.

In addition, an angle will need to be calculated between these two acceleration components. There will be no formulas that contain standard rotational variables such as ω and α.

Also, a new notation for the introduction of time will be introduced. For example, with an initial velocity of 1 m/s and a given tangential acceleration how long will it take (t) to reach a final velocity of 4m/s? The notation will be as follows (1m/s, 0s) and (4m/s,ts)

This alternative method eliminates the need for plotting variables in a two-dimensional graph with time on the horizontal axis. Except for the derivation of radial acceleration this paper will contain the full written section needed to be included in a standard Physics textbook with the appropriate number of fully worked example problems.

Introduction

This paper is not written from a research perspective. There was no collected student data from surveys as to the effectiveness of this third section. I have used parts of this paper in my physics and my engineering mechanics class “Dynamics”. While correcting exams and homework I have seen that most students have adopted taking curvilinear motion and representing it as linear segments as I presented it in class. This paper is written from a pure pedagogy perspective. In addition, a literature search of this topic provided no useful results. This approach works well with circles and for algebra and claculus-based physics classes. This only intended as an extension to straight line motion. Angular variables are then thoroughly covered in subsequent chapters.

The included third textbook section

Derivation of radial (centripetal) acceleration. It is not included here because it is not the focus of the new text-book section.

$$\textit{The Derived Equation}: \quad \mathrm{a_r} = \frac{\mathrm{v}^2}{\mathrm{r}}$$

Pure Circular Motion

We will consider two types of circular motion, Uniform and Non-Uniform circular motion. For both types of motion, we have a particle travelling on the circumference of a circle.

Let’s start with Uniform circular motion (UCM). UCM implies that the particles speed does not change with time as it travels around the circumference. See Figure 1 below.

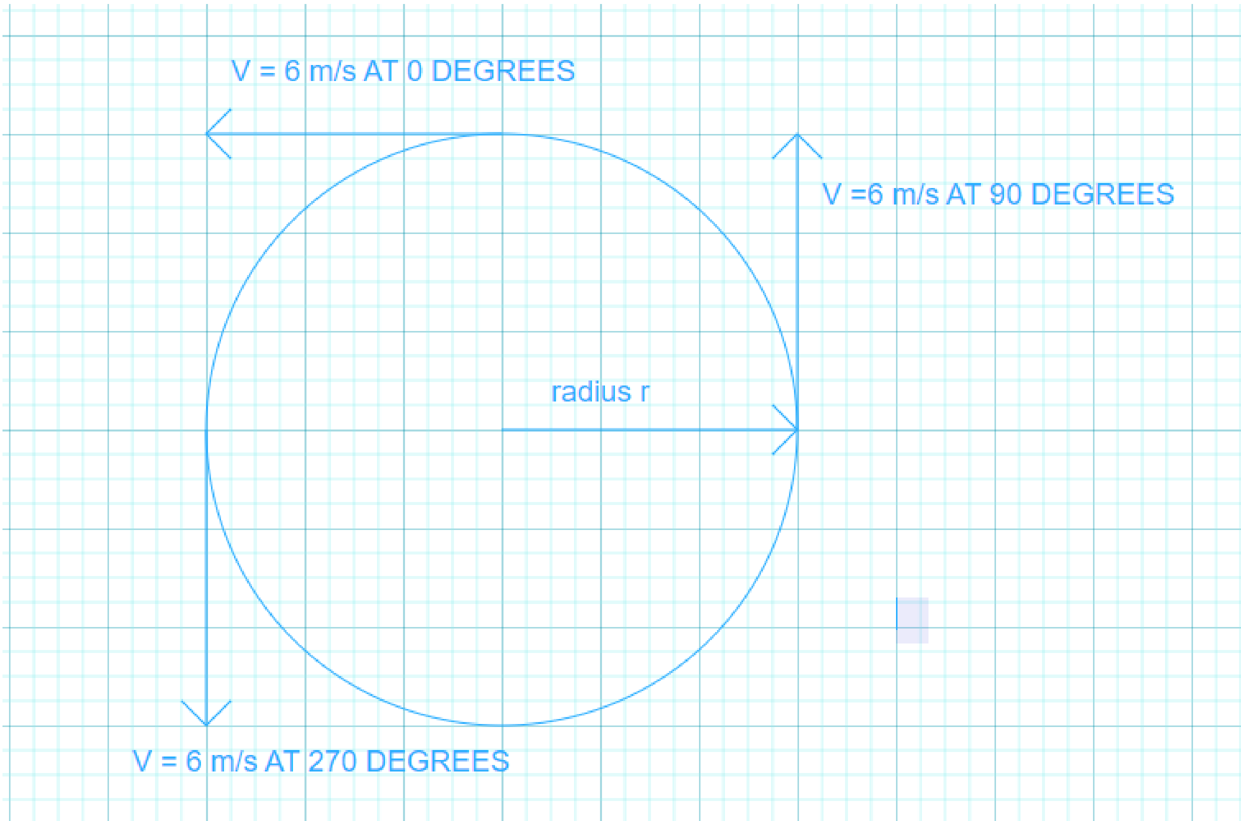

Figure 1

A velocity of 6 m/s was chosen at random. A velocity vector has two parts, magnitude, and direction. The magnitude part (6 m/s) is called speed, and the direction is given by an angle. As the particles moves around the circle its direction angle continuously changes as indicated in the figure 1 above. When there is a change in the velocity vector, due to either magnitude or direction changes we then have an acceleration. In this case, the direction angle changes and is called centripetal or radial acceleration. This radial acceleration ALWAYS points from the particle to the center of the circle. Note that the velocity vector is ALWAYS perpendicular to the radius of the circle. The velocity vector lies on a line that is Tangent to the circle as in Figure 2.

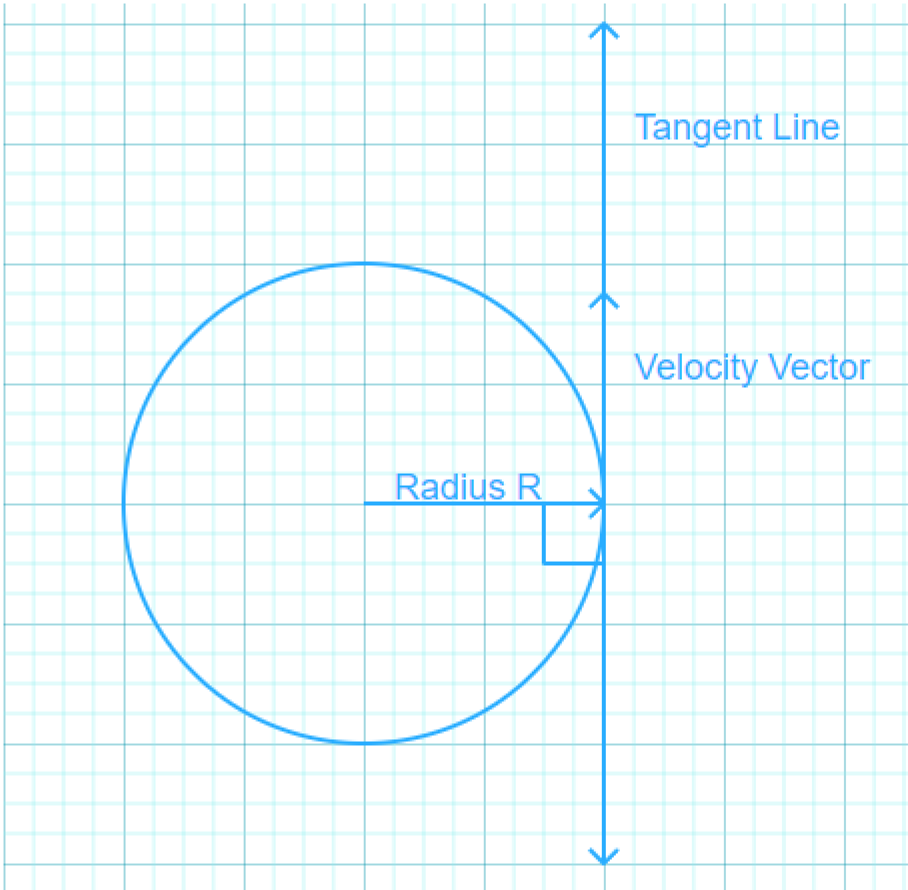

Figure 2

In this case the velocity vector is called Tangential Velocity.

$$v = Tangential\ Velocity$$

We may pick clockwise (cw) or counterclockwise (ccw) as the positive direction for particle motion. In this section we will choose ccw as positive. In addition, the tangential velocity vectors will also rotate ccw. The distance the particle travels in 1 revolution around the circle path is given by the following equation.

$$\mathrm{c = 2\pi R}$$

Where c = the circumference length of the circular path. We can imagine the circumstance length as being a straight-line segment of length 2πR. If we view circular motion in this way, we can use our One-Dimensional Kinematic Equations to describe the motion of the particle.

$$\mathrm{v = v_0 + at}$$

$$\mathrm{x = \ v_0 t + \frac{1}{2} a t^2}$$

$$v^2 = (v_0)^2 + 2ax$$

$$v = \text{The particle's final tangential velocity}$$

$$v_0 = \text{The particle's tangential velocity at time zero}$$

$$a = \text{the particle's linear acceleration}$$

In this section a is now called the particle's tangential acceleration

$$x = \text{the partile's final x coordinate}$$

Our convention will be to place the origin at the place where the particle starts its motion. Therefore x_o = 0m.

In addition, we can determine the number (n) of times the particle travels around its circular path with the following equation. A particle that travels once around its circular path covers a linear distance equal to the circumference length of the circular path. See Figure 3 for the positive n-axis

$$x = nc \text{ or } n = {}^{x}/_{c}$$

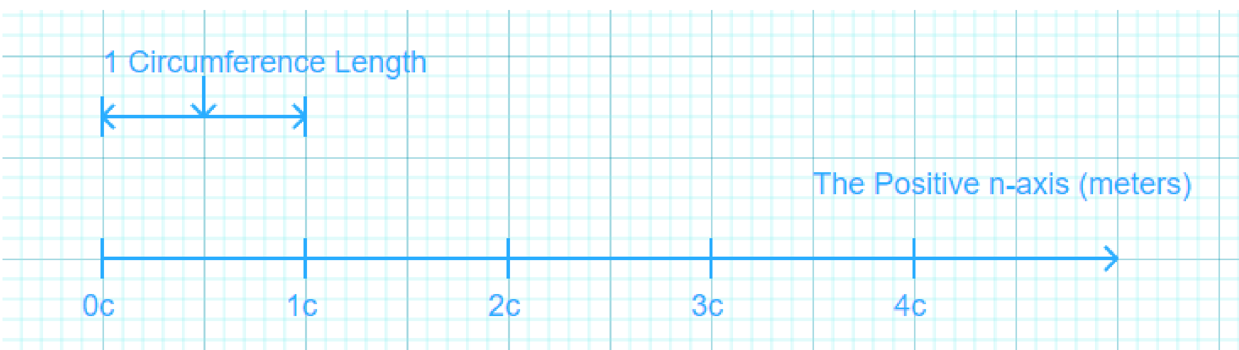

Figure 3

The period T is the time it takes the particle to travel once around its circular path. It is related to its tangential velocity by the following equation.

$$T = \frac{2\pi R}{v}$$

For UCM we have the following conditions for the particles motion around the circular path.

1.The magnitude (speed) of the tangential velocity vector is a constant.

2. The tangential acceleration a is zero.

3. The period T is a constant.

4. The radial acceleration a_r is a constant

Our kinematic equations reduce to the following for UCM:

$$v = v_0$$

$$x = v_0 t$$

Example 1) A particle is travelling ccw on a circular path of radius 10 m in UCM. Its constant speed is 20 m/s.

<u>What is the particle's radial acceleration?</u>

Equation Used: $a_r = \frac{v^2}{r}$

$$a_r = \frac{(20m/s)^2}{10m} = 40\ m/s^2$$

<u>What is the orientation of the particle's radial acceleration vector?</u>

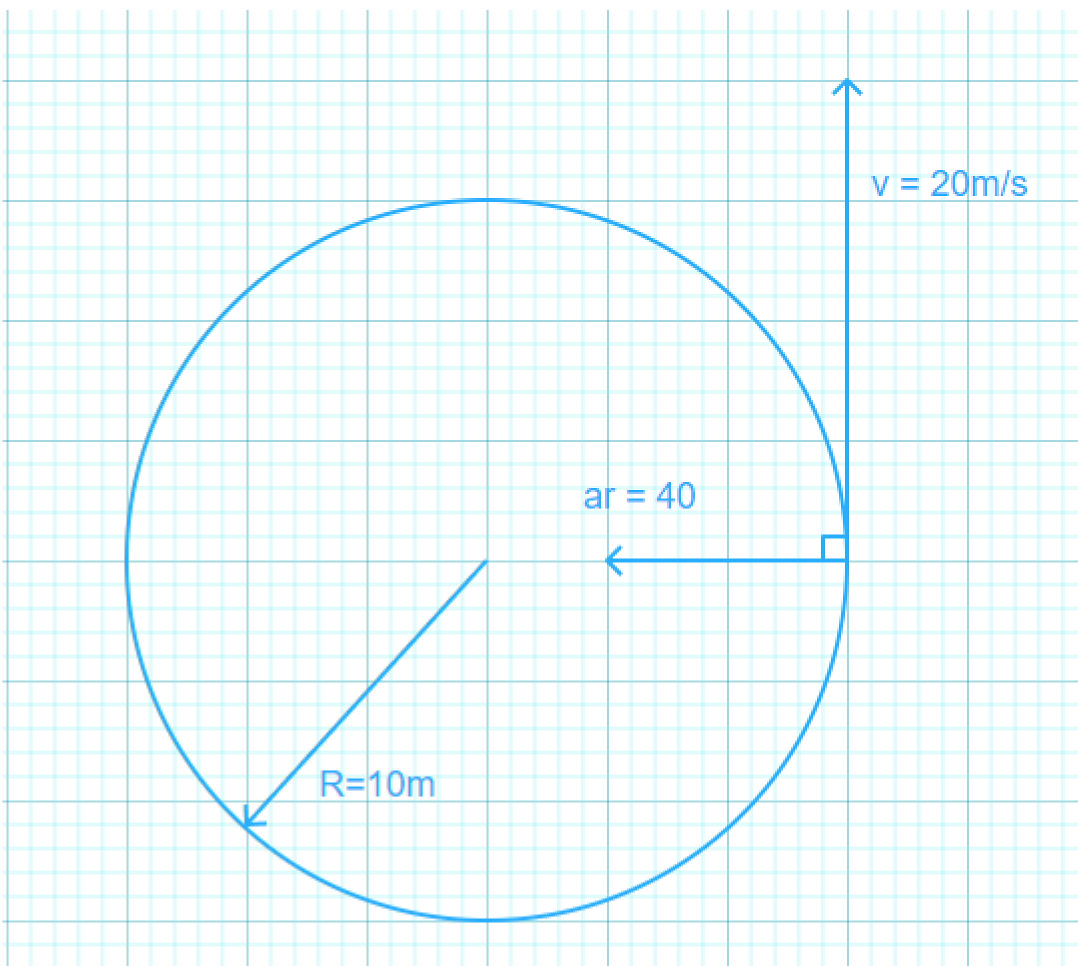

Figure 4

The position of the particle on the circle was randomly chosen. At any point on the circle the particle's radial acceleration points to the center of the circle. In addition, the tangential velocity vector is always perpendicular to the radial acceleration vector as shown in Figure 4.

<u>What is the particle's x-coordinate after 25 seconds?</u>

Equation Used: $\mathrm{x} = \mathrm{v_0 t}$

$$\mathrm{x} = \mathrm{v_0 t} = (20\ \mathrm{m/s})(25\ \mathrm{s}) = 500\mathrm{m}$$

<u>What is the particle's period T?</u>

Equation Used: $\mathrm{T} = \frac{2\pi \mathrm{R}}{\mathrm{v}}$

$$\mathrm{T} = \frac{2\pi(10\ \mathrm{m})}{20\ \mathrm{m/s}} = \pi\ seconds$$

<u>How many circumference lengths did the particle travel in 25 seconds?</u>

Equation Used: $x = nc \text{ or } n = {}^{x}/_{c}$

$$n = \frac{500m}{2\pi(10m)} = 7.96\ circumference\ lengths$$

Example 2) A particle is travelling in UCM and has a radial acceleration of 20 m/s^2. The radius of the circular path is 1.4 m.

<u>What is the particle's speed?</u>

Equation Used: $\mathrm{a_r} = \frac{\mathrm{v}^2}{\mathrm{r}}$

$$20\ m/s^2 = \frac{v^2}{1.4m}$$

$$v = \sqrt{\frac{28m^2}{s^2}} = 5.3\ m/s$$

What is the particle's period T?

Equation Used: $T = \frac{2\pi R}{v}$

$$T = \frac{2\pi(1.4\ \mathrm{m})}{5.3\ \mathrm{m/s}} = 1.7\ seconds$$

How long (t) does it take the particle to travel 6.4 circumference lengths?

First, we calculate the circumference.

Equation Used $c = 2\pi R$

$$c = 2\pi\ (1.4m) = 8.8\ m.$$

The value of 6.4 is n and we can then calculate the total distance travelled by the particle.

Equation Used: $x = nc\ or\ n = {}^{x}/_{c}$

$$x = The\ particles\ final\ x - coordinate = 6.4(8.8m) = 56.3\ m$$

Now using the particles calculated speed of 5.3 m/s we can calculate the time t.

Equation Used: $x = v_0 t$

$$56.3\mathrm{m} = (5.3\mathrm{m/s})\mathrm{t}$$
$$t = 10.6\ s$$

Non-Uniform Circular Motion

Now let's consider **non-UCM**. With UCM the particle's speed (v), period (T), radial acceleration (a_r) and tangential acceleration ($a = 0\ m/s^2$) are ALL constants (they do not change with time).

With **non-UCM** all 4 of the above variables change with time. Because the particles speed changes, there must be a non-zero tangential acceleration.

We will consider only problems with constant non-zero tangential acceleration. This will allow us to use the One-Dimensional Kinematic Equations to describe the motion of the particle.

The resultant acceleration vector (R) is the vector sum of the tangential and radial acceleration vectors. Its magnitude is given by the following equation:

$$R = \sqrt{(a_r)^2 + (a)^2}$$

Example 3) A particle travelling in non-UCM has a constant tangential acceleration of 34 m/s² and at a given point in time has a radial acceleration of 22 m/s².

<u>What is the resultant acceleration of the particle at that point in time?</u>

$$Equation\ Used: \quad R = \sqrt{(a_r)^2 + (a)^2}$$

$$R = \sqrt{(22\ m/s^2)^2 + (34 m/s^2)^2}$$

$$R = 40.5\ m/s^2$$

Example 4) A particle is travelling with a constant tangential acceleration of 10.1 m/s². The initial speed is 2.4 m/s, and the particle's final speed is 8.6 m/s. The radius of the circular path is 2.2 m.

<u>What is the particle's initial radial acceleration?</u>

$$Equation\ Used: \quad \mathrm{a_r} = \frac{\mathrm{v}^2}{\mathrm{r}}$$

$$a_r = \frac{(2.4\ m/s)^2}{2.2\ m} = 2.6\ m/s^2$$

<u>What is the particle's final radial acceleration?</u>

$$Equation\ Used: \quad \mathrm{a_r} = \frac{\mathrm{v}^2}{\mathrm{r}}$$

$$a_r = \frac{(8.6\ m/s)^2}{2.2\ m} = 33.6\ m/s^2$$

<u>How long does it take the particle to go from 2.4 m/s to 8.6 m/s?</u>

$$\text{Equation Used:} \quad \mathrm{v} = \mathrm{v_0} + \mathrm{at}$$

$$8.6 \text{ m/s} = 2.4 \text{ m/s} + 10.1 \text{m}/s^2(\text{t})$$

t = 0.6 s

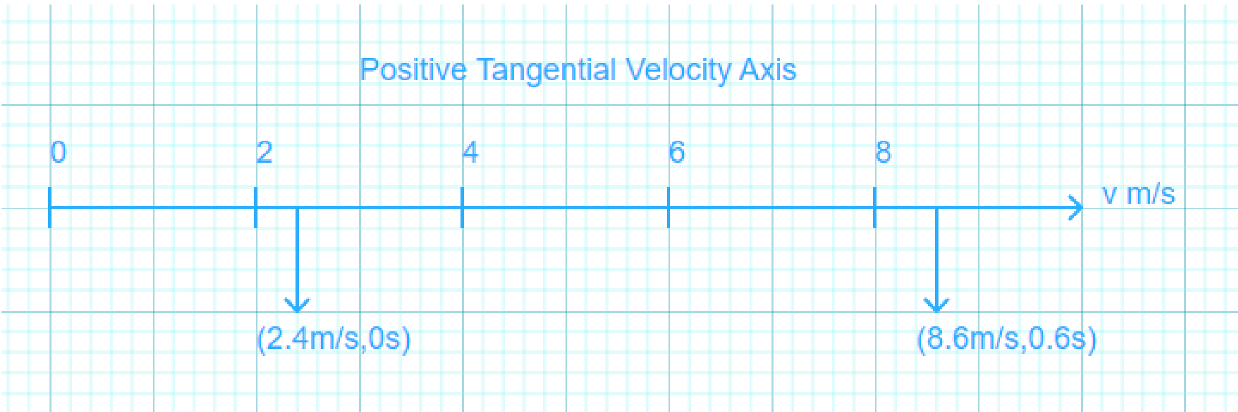

Figure 5

Figure 5 shows the graphical combination of tangential velocity and time on one-axis.

<u>How many circumference lengths (n) does it complete in going from 2.4 m/s to 8.6 m/s?</u>

First determine the final x-coordinate of the particle.

Equation Used: $\text{v}^2 = (\text{v}_0)^2 + 2\text{ax}$

$$(8.6 \text{ m/s})^2 = (2.4 \text{ m/s})^2 + 2(10.1 \text{ m}/s^2)\text{x}$$

x = 3.4 m

Equation Used: $x = nc \; or \; n = {}^{x}/_{c}$

$$n = \frac{3.4m}{2\pi(2.2m)} = 0.24 \; circumference \; lengths$$

<u>How long does it take to complete its first circumference length?</u>

Note that this is non-UCM, so it does not have a constant period T. Calculate the circumference length.

Equation Used $\text{c} = 2\pi\text{R}$

$$\text{c} = 2\pi(2.2\text{m}) = 4.4\pi \text{ m}$$

Now we could use the following equation to solve for the first period time T_1.

Equation Used: $x = v_0 t + \frac{1}{2}at^2$

$$4.4\pi \text{ m} = (2.4\ m/s)\text{t} + \frac{1}{2}(10.1\ \text{m}/s^2)(T_1)^2$$

Instead of solving this quadratic equation we could do the following which is mathematically easier. First solve the equation below for the particle's tangential velocity at x =4.4π m.

Equation Used : $v^2 = (v_0)^2 + 2ax$

$$v^2 = (2.4\ \text{m/s})^2 + 2(10.1\text{m}/s^2)(4.4\pi\ \text{m})$$

$$v = 16.9\ \text{m/s}$$

Now we can use that calculated value in the following equation.

Equation Used: $v = v_0 + at$

$$16.9\ \text{m/s} = (2.4\ m/s) + (10.1\ \text{m}/s^2)T_1$$

$$T_1 = 1.4\ \text{s}$$

Because the particle is accelerating the second revolution time T_2 will take less than 1.4 s.

<u>What is the magnitude and direction of total initial Resultant Acceleration of the particle?</u>

Equation Used: $R = \sqrt{(a_r)^2 + (a)^2}$

$$R = \sqrt{(2.6m/s^2)^2 + (10.1m/s^2)^2}$$

$$R = 10.4\ \text{m/s}^2$$

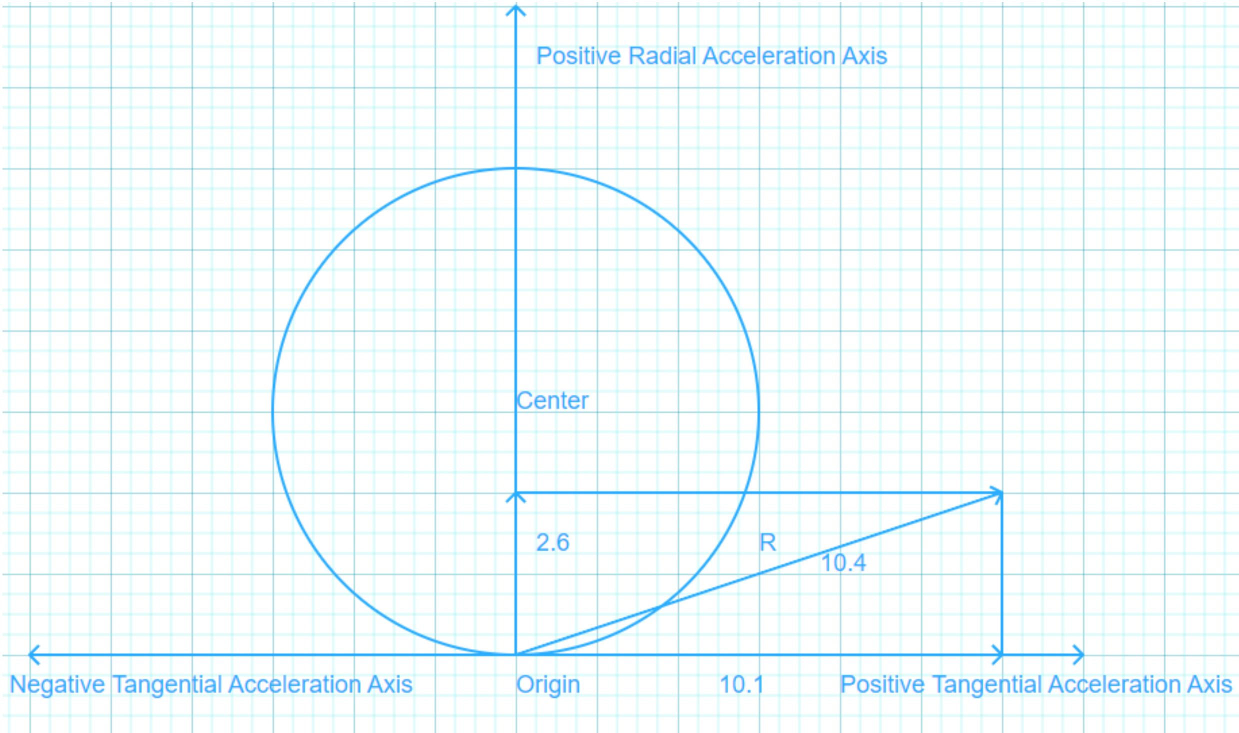

Figure 6

For simplicity purposes we will freeze the motion of the particle at the moment of interest and rotate the circle until the particle is at the origin as in the Figure 6 above. We will then generate the positive a_r -axis and the positive and negative a-axes as shown above in Figure 6. The resultant vector R will then be in the first quadrant if the particle is accelerating or in the second quadrant if the particle is decelerating.

<u>What angle Ɵ does R make with the positive a axis?</u>

From Figure 6 we can see that the following equation can be written:

$$\Theta = \tan^{-1}\frac{2.6}{10.1} = 14.4°$$

Example 5) A particle with an initial velocity 32 m/s travels along a linear path for 6m. It has a constant tangential deceleration of 8 m/s^2. At the end of the 6m linear path it enters into a circular path with radius of 2m and completes 4 circumference lengths. See Figure 7

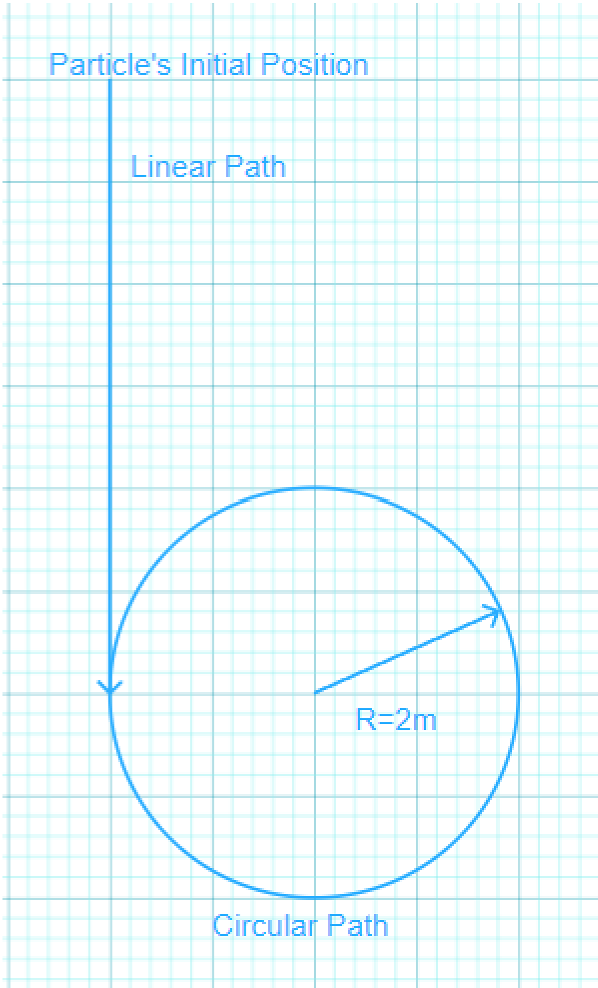

Figure 7

What is the total distance travelled by the particle?

The total distance travelled is composed of the linear length (6m) and four circumference lengths. This is also the final x-coordinate of the particle.

Final x-coordinate = 6m + 4[2π(2m)] = 56.3m

What is the particle's final velocity?

We will use the following kinematic equation.

Equation Used: $v^2 = (v_0)^2 + 2ax$

$$v^2 = (32m/s)^2 + 2(-8m/s^2)(56.3m)$$

v = 11.1 m/s

What is the particle's radial acceleration when it first enters the circular path?

We must first calculate the particle's velocity at the end of the 6m linear path.

$Equation\ Used$: $\mathrm{v}^2 = (\mathrm{v}_0)^2 + 2\mathrm{ax}$

$$\mathrm{v}^2 = (32m/s)^2 + 2(-8\mathrm{m}/s^2)(6\mathrm{m})$$

v =30.5 m/s

$Equation\ Used$: $\mathrm{a_r} = \dfrac{\mathrm{v}^2}{\mathrm{r}}$

$$\mathrm{a_r} = \frac{(30.5\ \mathrm{m/s})^2}{2\mathrm{m}} = 464\ m/s^2$$

What is the particle's final Resultant acceleration?

We must use the particle's final tangential velocity (11.1 m/s) to calculate the final radial acceleration.

$Equation\ Used$: $\mathrm{a_r} = \dfrac{\mathrm{v}^2}{\mathrm{r}}$

$$\mathrm{a_r} = \frac{(11.1\mathrm{m/s})^2}{2\mathrm{m}} = 61.6\ m/s^2$$

Now we must use this value to calculate R.

$Equation\ Used$: $R = \sqrt{(a_r)^2 + (a)^2}$

$$R = \sqrt{(61.6m/s^2)^2 + (-8m/s^2)^2}$$

R= 62.1 m/s^2

Now let's look at the direction of the R vector from Figure 8 below.

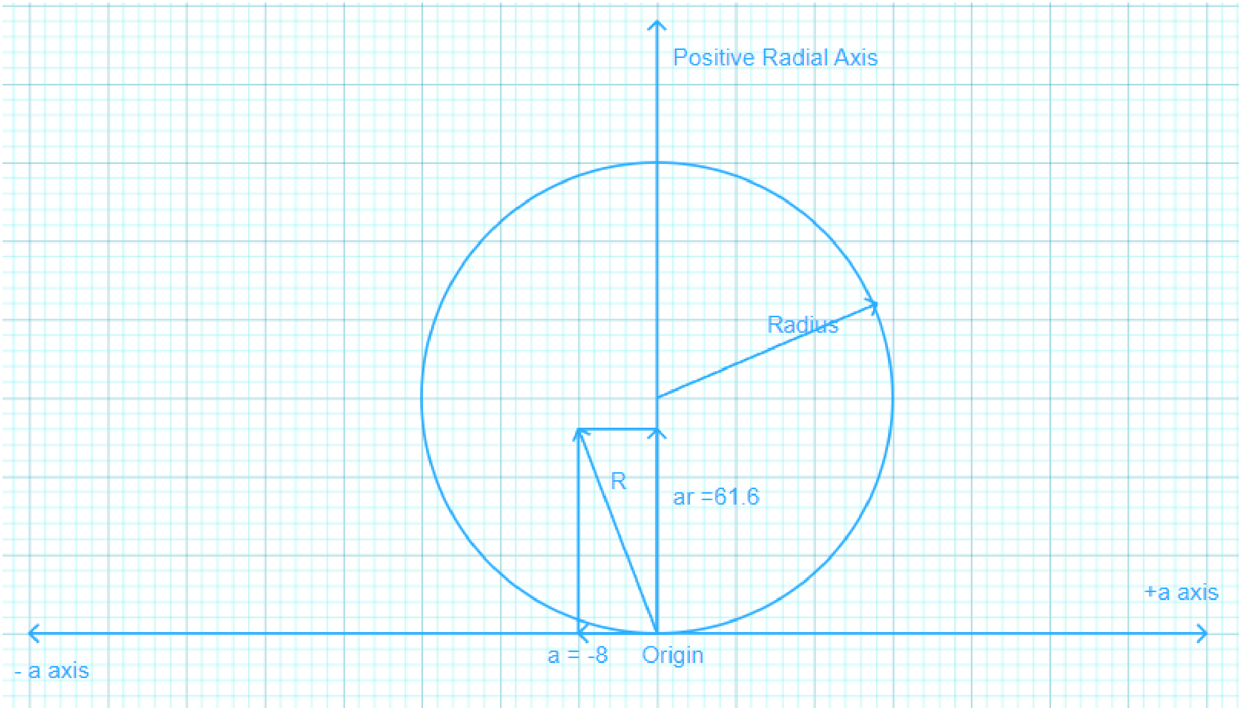

Figure 8

Once again for simplicity purposes we will freeze the motion of the particle at the moment of interest and rotate the circle until the particle is at the origin as in figure 8 above. We can see from the figure that that the Resultant acceleration vector R is in the second quadrant. This indicates that the particle is decelerating.